B.Sc. Semester – IV

US04CSTA21

Unit – IV

Statistical Quality Control (SQC)

Introduction:

After the industrial revolution the world has progressed enormously in the field of industry. In the modern age of industrial development, there is competition, practically in every branch. Hence one of the primary needs of the top management in any production organization is to ensure that the products manufactured by them are of good quality. The only method available in the past was to inspect the items produced at various stages of production and also before the products leave the factory. In the present age of competition different manufacturers put the same type of product in the market. Hence the basic aim of any manufacturer is to see that his product should compete with other products from the view point of quality and price. So the main objective of a manufacturer should be to maintain a uniform standard of his products. To maintain the quality of a product at a standard level and to decrease the cost of production, the technique of quality control is used. The technique of quality control is a continuous process in which the need for action is identified at the earliest possible stage.

These techniques were introduced by Dr. W.A. Shewhart during the second world war. He developed the technique of quality control to know whether the products are of standard quality or not.

Quality and Quality control:

It is very easy to understand that the primary aim of any manufacturer is to see that the products manufactured by him should be acceptable to the customers. Once a particular product becomes acceptable to the customers, he has to be vigilant about maintaining the same standard of quality. In statistical quality control, the term quality does not mean the best one, but it is related to some specified standards. Before starting manufacturing a product a manufacturer always decides certain standards of his products by keeping in mind the customer's liking, the resources available, the cost etc. and if the product satisfies the predetermined standards, it can be said that the quality is maintained. This does not mean that products of better cannot be produced. If products of better quality are to be manufactured then cost will increase and the customer may not afford them. Hence it is advisable to maintain the consistency in quality standard. The need for quality control arises because in any repetitive process a certain amount of variability is inevitable. No doubt, we have today, sophisticated machines; handled by highly skilled operators and raw material of progressively better quality, but small variations from item to item will still be there. No two items will be identical in all respects. Variability in the products is inherent characteristics of any manufacturing process. This basic process of variability of the production system is the result of the minor variations in the various components of the system, namely, the machines, the raw materials, the operators, or change in time. At times there may be significant variations in the quality of the products. These may be due to major defects in the machines, carelessness of operators etc. these variations should be considered serious and their causes should be detected and removed. The main purpose of SQC is to separate out these two types of causes of variations.

Variations in quality:

If any two products are examined from a production process they cannot be identical in all respect. Some sort of variation in the two items is bound to be there. In fact, it as an integral part of any manufacturing process. If two screws are taken from the production of a factory it is never possible that they are equal in

all respect. There is bound to be a minute difference either in the length or in the diameter of screws. This difference in the characteristic of a product is known as variation. If two products manufactured on the same machine by the same operator and under similar conditions are closely examined, we will find some difference in two. If efforts are made to search for the causes of these variations, many times it is vain. At times the variations may be due to the substandard quality of raw material, careless on the part of operators, faults in the machinery system etc. the causes of variations due to these factors can be traced out and the quality of the products can be improved. The variations in the quality of products can therefore be divided into two parts:

(i) **Variations due to chance causes**

(ii) Variations due to assignable causes

(i) Variations due to chance causes:

Sometimes the variations in the quality of products may not be due to any defect in the machine, carelessness of operator etc. but occur due o chance. These variations result from many minor causes and behave in a random manner. The products may produced on the same machine, by the same operator and under similar conditions still however such minor variations in are due to combined influence of several minor causes of variations. The variations due to these complex set of causes are negligible and inevitable. These variations are due to chance causes. The variations due to chance causes follow a definite statistical law.

(ii) Variations due to assignable causes:

Sometimes we observe that variations in the products manufactured by a production process. These variations may be due to some non-random causes like improper machine setting, inexperienced operators, poor quality of raw materials etc. the variations due to these causes , are known as assignable variations. The variations due to assignable causes are serious in nature. The causes of these variations should be detected and should be removed.

We shall now discuss about different assignable causes of variations.

(1) Difference in the quality of raw material:

In any manufacturing process the changes in the quality of the raw material result into the changes in the quality of the products. If the copper of substandard (lower) quality is used, the wires manufactured will not be of good quality. The variation due to inferior quality of raw material can be called assignable variation. The causes of such variations can be detected, removed and the quality can be improved.

(2) Difference in machines:

We know that in any manufacturing process the same type of product is manufactured by number of different machines. Some machines may be very old while some may be quite new. It is obvious that products from a new machine will be better than those from an old one. Hence there will be variations in the products due to different machines. The variations in the products due to difference in machine are regarded as significant variations and they can be found out. These variations are due to assignable causes.

(3) Difference in operators:

Generally the same machine is used by different operators. Some of them may be highly skilled and efficient while some may be untrained and inefficient. Because of the differences among operators the quality of the products can also suffer. Such variations are also due to assignable causes. The

causes of such variations can be detected, removed and the quality can be improved.

(4) Difference of time:

The production in most of the industries is carried out in different shifts. It is quite obvious that the workers in the morning shift are fresh and do their work more enthusiastically, while those in the night shift may be tired and less enthusiastic. The quality of product may not be same in two different shifts. Even in the same shift, the same worker may be more attentive in his work in the beginning than at the end. The variations in the quality can also be due to this factor. Production in textile industry can also be affected by weather condition. There can be variations in the quality of because of seasons. Changes in time can thus be considered assignable causes of variations in quality.

Thus there are two types of causes of variations in any manufacturing process. The main purpose of SQC is to derive statistical methods for separating out assignable causes of variations from chance causes. The statistical technique used for this purpose is control chart analysis.

Theory of Runs:

We have seen that in any control chart if one or more points fall outside the control limits, the process is said to be out of control, and the presence of assignable causes of variations is suspected in the process. The process can be brought under control by removing such assignable causes of variations. If all sample points fall within two control limits, the process is said to be under control, and it may be concluded that only the chance causes of variations are present in the process. It is simple to understand that the probability of any point falling on either side of the central line is same. i.e. a point fall on the upper side of the central line is $\frac{1}{2}$ and that a point falls on the lower side of the central line is also $\frac{1}{2}$ this situation may not be regarded as random. The probability of two consecutive points falling on the upper side of the central line is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Similarly the probability of 7 consecutive points falling on the upper side of the central line is $\left(\frac{1}{2}\right)^7 = \frac{1}{128}$. In the same way the probability of 7 consecutive points falling on the lower side of the central line is $\left(\frac{1}{2}\right)^7 = \frac{1}{128}$. Thus the probability that 7 consecutive points fall on either side of central line is $\frac{1}{128} + \frac{1}{128} = \frac{1}{64}$. This probability is very less.

The falling of consecutive points on the same side of central line is said to be run. If they fall on the upper side of the central line is said to be "run above" and if they fall on the lower side of the central line, it is said to be "run below". If run above or run below is observed in a control chart, the shift in average may be suspected. If the run above is observed the shift is average on the upper side is suspected. If the run below is observed the shift in average on the lower side may be suspected.

Uses of S.Q.C:

In the modern age of industrial development maintaining the quality of the product is very essential and inevitable for any management. There are various uses of SQC in industry. Some of them are as follows:

(a) to define the standards of the process.

(b) to attain such standards.

(c) to maintain the standards.

(d) to detect assignable causes of variations and to remove them.

(e) the efficiency of workers may be improved by giving special training.

(f) the cost of production can be reduced.

Charts for variables:

The charts for the measurable quality characteristic of the products are known to as charts for variables. Certain characteristic like length of a screw, weight of an article, diameter of a ring can be measured. The charts used to know whether the desired standard of these products are maintained or not, are known as variable chart. The main variable charts are $\overline{X}$ and R charts.

$\overline{X}$ chart:

The measurable characteristic of the product is denoted by X. m be the no. of samples (no. of subgroups) and the sample of size n are drawn at more or less regular interval of time. These samples are known as subgroups, and for each subgroup the values of mean a $\overline{X}$ nd range R are obtained. If the distribution of variable X is normal with mean μ and s.d σ then the distribution of $\overline{X}$ is also normal with mean μ and s.d $\frac{\sigma}{\sqrt{n}}$. Moreover Dr. Shewhart has shown that even if, the distribution of X is not normal, and the number of units in each subgroup is 4 or more, then also the distribution of $\overline{X}$ is also normal with mean μ and s.d $\frac{\sigma}{\sqrt{n}}$

The control limits for $\overline{X}$ chart are:

LCL = $\overline{\overline{X}} - A_2\overline{R}$

CL = $\overline{\overline{X}}$

UCL = $\overline{\overline{X}} + A_2\overline{R}$

The value of A_2 is also available from the statistical table.

R Chart:

For R chart the values of range of range R is obtained for each subgroup taken at a regular interval. The range is the difference between the highest and lowest observations of a subgroup. From the values of R obtained from m subgroups, the average $\overline{R}$ is calculated. i.e. $\overline{R} = \frac{\sum Ri}{m}$

The control limits for $\overline{R}$ chart are:

LCL = $D_3\overline{R}$

CL = $\overline{R}$

UCL = $D_4\overline{R}$

The value of D_3 *and* D_4 *are* constants which depend upon subgroup size n and are available in the statistical table.

(iii) Construction of $\overline{X}$ and $\overline{R}$ charts:

Samples of equal size are drawn at regular interval of time from production process and from each sample the values of mean $\overline{X}$ and range R are obtained. From these values $\overline{\overline{X}}$ and $\overline{R}$ are calculated as follows:

$$\overline{\overline{X}} = \frac{\sum X}{m} \text{ and } \overline{R} = \frac{\sum Ri}{m}$$

From the values of $\overline{\overline{X}}$ and $\overline{R}$ the control limits are obtained as follows.

For $\overline{X}$ chart :

LCL = $\overline{\overline{X}} - A_2\overline{R}$

CL = $\overline{\overline{X}}$

UCL = $\overline{\overline{X}} + A_2\overline{R}$

For R Chart:

LCL = $D_3\overline{R}$

CL = $\bar{R}$

UCL = $D_4\bar{R}$

For $\bar{X}$ chart the subgroups numbers are taken on x-axis and the values of $\bar{X}$ are taken on y-axis. The central line and both the control limits are drawn parallel to x-axis. The values of $\bar{X}$ for each sample is plotted on the graph paper on joining these points in order to get $\bar{X}$ chart.

For R chart the values of R are taken on y-axis and the central line, LCL and UCL are drawn parallel to x-axis. The values of R obtained from each sample is plotted on the graph paper on joining these points in order to get R chart.

(iv) Conclusion from $\bar{X}$ and $\bar{R}$ charts:

(a) From $\bar{X}$ chart:

If all the points on $\bar{X}$ chart fall within the control limits and if they are randomly distributed on both the sides of the central line, then the process is said to be under statistical control. This shows that only chance causes are present in the process.

If one or more points fall outside the control limits, the process is said to be out of control with respect to average. This indicates the presence of assignable causes in the process.

(b) From R chart:

If all the points on R chart fall within the control limits and if they are randomly distributed on both the sides of the central line, then the process is said to be under statistical control. This shows that only chance causes are present in the process.

If one or more points fall outside the control limits, the process is said to be out of control with respect to range. This indicates the presence of assignable causes in the process.

(c) From $\bar{X}$ and R chart simultaneously:

For better control of the process it is preferable to draw both $\bar{X}$ and R charts on the same graph paper. If on $\bar{X}$ chart one or more points fall outside the control limits and on R chart all the points fall within the control limits, then it can be said that the process is not under control with respect to average, but it is under control with respect to range. This indicates that the variation between the subgroups is significant, while that within the subgroup is insignificant.

It can also be happen that process may be under control with respect to mean, but may not be under control with respect to range. So while deciding about the state of control in any production process, it is necessary to study about the control both from average view point and the variability view point.

Charts for Attributes:

$\bar{X}$ and R charts are charts for variables. i.e. for quality characteristic that can be measured. Some of the quality characteristic cannot be measured, but we can decide whether an item possesses a particular characteristic or not. For example, on inspection of an electric bulb or a bottle, we can decide whether it is good or bad. In such cases each product is classified either as defective (non-conforming) or non-defective (conforming) depending upon whether the item satisfies certain standards or not. The charts based upon quality characteristic i.e. attributes are known as attribute chart.

Sometimes only one unit of product may have number of defects and charts prepared after inspecting many such units are known as charts for attributes. The following are main attribute charts.

(i) p – chart or fraction defective chart

(ii) np – chart or number of defectives chart

(iii) C – chart or number of defects per unit chart.

(a) (i) p and np chart:

Sometimes the items can be classified as defective or non-defective. If a sample of size n is taken from a production process, and on inspection of each item, d items are found to be defectives in the sample, then the fraction defective p of sample is

Fraction defective $p = \frac{d}{n} = \frac{Number\ of\ defective\ items\ in\ the\ sample}{Number\ of\ items\ inspected\ in\ the\ sample}$

If different samples of size n are drawn at more or less regular intervals of time for each sample the value of p can be obtained. To know whether the process is under control or not with respect to fraction defective p, we should study the distribution of p. The distribution of p found to be Binomial distribution.

The control limits for p – chart:

LCL= $\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

CL= $\bar{p}$

UCL= $\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

Sometimes instead of fraction defective p the total number of defectives np of a sample is taken and the chart for number of defectives is obtained. This chart is known as np chart. If d = np is the total number of defectives observed in a sample of size n, then the distribution of d is also Binomial.

The control limits for np – chart:

LCL= $n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$

CL= $n\bar{p}$

UCL= $n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$

(ii) Construction of p and np chart:

m samples each of size n are drawn at regular interval of time from a production process and for each sample the number of defectives d is found out. From it, the fraction defective $p = \frac{d}{n}$ **is obtained. The average** $\bar{p}$ **of fraction defectives of m samples is found out. i.e.** $\bar{p} = \frac{\sum p}{m}$ **.**

The control limits for p –chart are

LCL= $\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

CL= $\bar{p}$

UCL= $\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

Now taking the sample numbers on x-axis and fraction defectives on y-axis, the control limits are drawn parallel to x-axis. The values of p for each sample are plotted on the graph paper. On joining these points joining order, we obtain p chart.

If instead of p – chart, np chart is to be drawn it is not necessary to find fraction defectives p for each sample. Instead of that the number of defectives d = np is obtained from each sample. From different values of np, the average number of defectives $n\bar{p}$ **is found out. i.e.** $n\bar{p} = \frac{\sum d}{m}$ **where d = np.**

The control limits for np – chart:

LCL= $n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$

CL= $n\overline{p}$

UCL= $n\overline{p}+3\sqrt{n\overline{p}(1-\overline{p})}$

Now taking the sample numbers on x-axis and number of defectives on y-axis, the control limits are drawn parallel to x-axis. The values of np for each sample are plotted on the graph paper. On joining these points joining order, we obtain np chart.

(iii) Conclusion from p and np chart:

(a) **In p or np chart if all points fall within the control limits and if they are not on the same side of the central line then the production process is said to be under control. It indicates the presence of only chance causes in the process.**

(b) **If one or more points fall above UCL the process is said to be out of control which indicates the presence of assignable causes of variations in the process.**

(c) **In p or np chart if LCL is negative then it is taken as zero.**

(d) **If one or more points fall below LCL then the general interpretation is that process is out of control. In fact this situation should be welcomed. Sample for which the points fall below LCL, the fraction defective is less and its quality may be considered better. Such points are known as "Low Spots". If low spots are found in control chart then they can be interpreted in the following way:**

(i) **The quality of the samples of low spots is superior and attempts should be made to maintain that standard.**

(ii) **We should try to find out whether in inspection of items of such samples any error is committed or not.**

Uses of p and np chart:

The following are some of the uses of p and np charts:

(i) When some of the points fall above UCL then the presence of assignable causes is suspected. These causes of variations should be detected and removed.

(ii) If some of the points fall below LCL then it may be regarded as a sign of improvement in the quality and attempts should be made to maintain the standard.

Difference between p and np charts

p - chart	np - chart
(1) p chart if chart for fraction defective	(1) np chart is a chart of number of defectives
(2) the calculations are difficult in p chart	(2) calculations are relatively easy
(3) p chart can be drawn even if all the samples are not of equal size.	(3) np chart can be drawn only when the samples are of equal size.

(b) C chart:

We have discussed p chart for fraction defective and np chart for number of defectives. In many cases it is more convenient to work with the number of defects per unit than the fraction defectives. For example, a radio set can have defects at innumerable points, but in practice we hardly find one or two defects in it. In such a case it is more convenient to count the number of defects per radio set rather than classifying the set as defective or non-defective one.

The number of defects in a unit taken from a production process is denoted by C and the control chart based on different values of C is known as C chart. Here we should note the difference between a defective item and a defect in an item. A defective item is an item which fails to conform certain specification or standard, while an item lack of conformity at one or more points are known as defects.

There can be many defects in a defective item. When number of defects per unit can be counted, C chart can be drawn. In manufacturing an item like radio, there are innumerable places where defects can occur, but the probability of a defect is very small. Hence in such cases Poisson distribution is applicable.

The control limits for C chart:

LCL= $\overline{C} - 3\sqrt{\overline{C}}$

CL= $\overline{C}$

UCL= $\overline{C} + 3\sqrt{\overline{C}}$

Construction of C chart:

m samples each of one unit are taken from a production process. The number of defects C in each unit is found out. The average number of defects $\overline{C}$ is obtained from m sampling units. $\overline{C} = \frac{\sum C}{m}$. The control limits for C chart will be as under:

LCL= $\overline{C} - 3\sqrt{\overline{C}}$

CL= $\overline{C}$

UCL= $\overline{C} + 3\sqrt{\overline{C}}$

Now taking the sample numbers on x-axis and number of defects on y-axis, the control limits are drawn parallel to x-axis. The values of C (no. of defects) for each sample are plotted on the graph paper. On joining these points joining order, we obtain C chart.

Conclusions from c chart:

The following conclusions can be drawn from C chart.

(a) **In all the points fall within the control limits and if they are not on the same side of the central line then the production process is said to be under control. It indicates the presence of only chance causes in the process.**

(b) **If one or more points fall above (outside) UCL the process is said to be out of control which indicates the presence of assignable causes of variations in the process. These causes of variations should be found out and removed.**

(c) **In C chart if LCL is negative then it is taken as zero.**

(d) **If one or more points fall below LCL then the general interpretation is that process is out of control. In fact this situation should be welcomed. Sample for which the points fall below LCL, the fraction defective is less and its quality may be considered better. Such points are known as "Low Spots". If low spots are found in control chart then they can be interpreted in the following way:**

(i) The quality of the samples of low spots is superior and attempts should be made to maintain that standard.

(c) We should try to find out whether in inspection of items of such samples any error is committed or not.

Uses of C chart:

C chart can be used in controlling the production processes.

(i) For the number of defects in electronic items like radio, TV etc.

(ii) For the number of faults in pipers or wires of given length etc.

Difference between variable charts and attribute charts

Variable charts	Attribute charts
(1) These charts can be used when the quality	(1) These chart can be used when the quality

characteristic is measurable.	characteristic is not measurable but when it can be decided whether the product possesses a particular characteristic.
(2) $\bar{X}$ and R charts are used for variables	(2) p, np and C charts are used for attributes
(3) More time and money required in these charts	(3) less time and money required in these charts
(4) in destructive testing, these charts can be used	(4) In destructive testing these charts cannot be used

Examples:

1 **A factory produces 50 cylinders per hour. Samples of 4 cylinders are taken at random from the production at every hour and the diameters of cylinders are measured. Draw $\bar{X}$ and R charts and decide whether the process is under control or not.**

Sample no.	Diameter of cylinders (in 0.01 cm)			
	1	2	3	4
1	230	238	242	250
2	220	230	218	242
3	222	232	236	240
4	250	240	230	225
5	228	242	235	225
6	248	222	220	230
7	232	232	242	242
8	236	234	235	237
9	231	248	251	271
10	220	222	224	231
11	222	233	244	255
12	272	262	265	225
13	218	268	274	250
14	214	218	252	262
15	260	262	265	263

Solution:

Here m = no. of samples = no. of subgroups = 15

n = sample size = 4

$$\bar{\bar{X}} = \frac{\sum \bar{X}}{m} = \frac{3591.75}{15} = 239.45$$

$$\bar{R} = \frac{\sum R}{m} = \frac{385}{15} = 25.67$$

sample no.	Diameter of Cylinders (in 0.01 cm)				Mean $\bar{X_i}$	Range R_i
	1	2	3	4		
1	230	238	242	250	240	20
2	220	230	218	242	227.5	24

3	222	232	236	240	232.5	18
4	250	240	230	225	236.25	25
5	228	242	235	225	232.5	17
6	248	222	220	230	230	28
7	232	232	242	242	237	10
8	236	234	235	237	235.5	3
9	231	248	251	271	250.25	40
10	220	222	224	231	224.25	11
11	222	233	244	255	238.5	33
12	272	262	265	225	256	47
13	218	268	274	250	252.5	56
14	214	218	252	262	236.5	48
15	260	262	265	263	262.5	5

$A_2 = 0.729 \ D_3 = 0 \ D_4 = 2.282$

Control limits for $\overline{X}$ chart:

LCL = $\overline{\overline{X}} - A_2\overline{R}$ = 220.74

CL = $\overline{\overline{X}}$=239.45

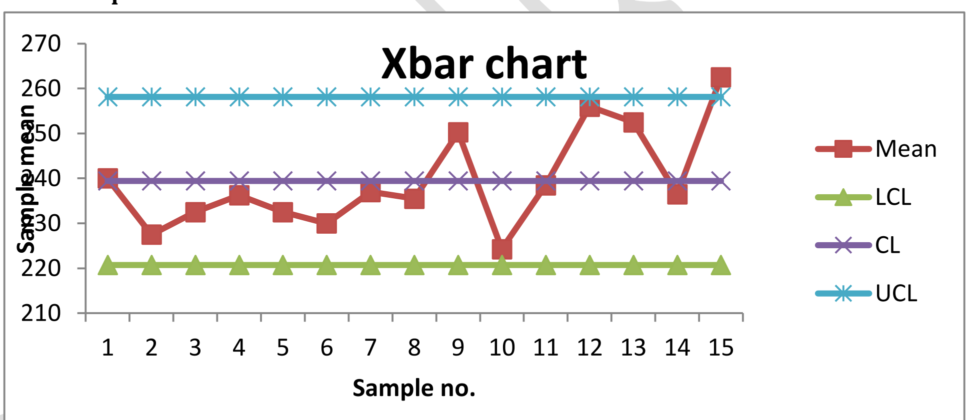
UCL = $\overline{\overline{X}} + A_2\overline{R}$ =258.16

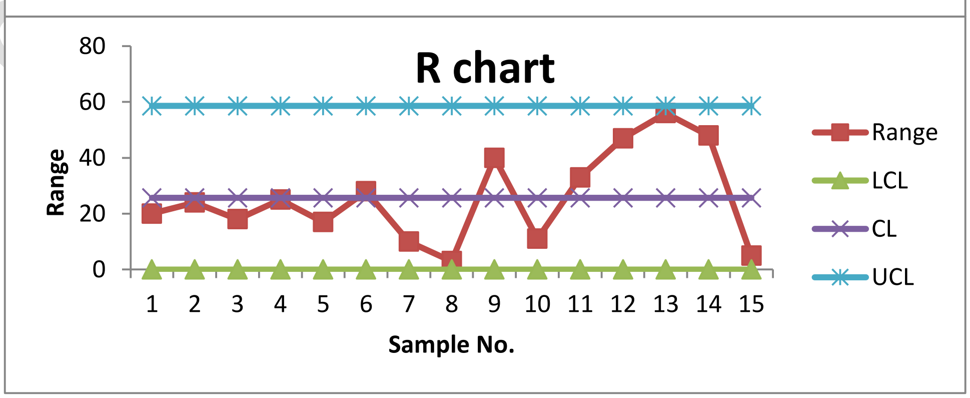
Control limits for R Chart:

LCL = $D_3\overline{R}$ = 0

CL = $\overline{R}$ = 25.67

UCL = $D_4\overline{R}$ = 58.57





Conclusions: The value for $\overline{X}$ for sample no. 15 falls outside the upper control limit. Hence the process is not under control with respect to average. All the sample points of R chart fall within the control limits the process may be regarded as under control from the variability point of view. Hence there is a significant variation between samples and insignificant variations within samples.

2 **Draw $\overline{X}$ and R charts and decide whether the process is under control or not.**

Sample no.	1	2	3	4	5	6	7	8	9	10
$\overline{X}$	12.8	13.1	13.5	12.9	13.2	14.1	12.1	15.5	13.9	14.2
R	2.1	3.1	3.9	2.1	1.9	3.0	2.5	2.8	2.5	2.0

Given that sample size n = 5.

Solution:

Control limits for $\overline{X}$ chart:

LCL = $\overline{\overline{X}} - A_2\overline{R}$ = 12.04

CL = $\overline{\overline{X}}$=13.53

UCL = $\overline{\overline{X}} + A_2\overline{R}$ =15.02

Control limits for R Chart:

LCL = $D_3\overline{R}$ = 0

CL = $\overline{R}$ = 2.59

UCL = $D_4\overline{R}$ = 5.48

3 **The following table gives the information regarding life hours of 5 LED lamps of 10 different samples. Draw $\overline{X}$ and R charts and decide whether the process is under control or not.**

Sample no.	1	2	3	4	5	6	7	8	9	10
$\overline{X}$	3290	3180	3350	3370	3280	3240	3260	3410	3310	3510
R	360	210	50	100	50	400	500	200	300	600

Solution:

Control limits for $\overline{X}$ chart:

LCL = $\overline{\overline{X}} - A_2\overline{R}$ = 3159.34

CL = $\overline{\overline{X}}$=3320

UCL = $\overline{\overline{X}} + A_2\overline{R}$ =3480.66

Control limits for R Chart:

LCL = $D_3\overline{R}$ = 0

CL = $\overline{R}$ = 277

UCL = $D_4\overline{R}$ = 584.47

4 **Samples are drawn at regular interval of time from a production process. From 25 samples $\sum \overline{X} = 365$ and $\sum R = 30$ are found out. Obtain the control limits for $\overline{X}$ and R charts.**

Solution:

For $\overline{X}$ chart: LCL = 13.74, CL=14.6, UCL=15.46

For R chart: LCL=0, CL=1.2, UCL=2.74

5 **From a pharmaceutical company samples of 400 bottles were taken daily for 15 days. The number of defective seals in these bottles is given below. Draw p chart or draw control charts for fraction defectives and comment on it**

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

No. of defective seals	28	18	40	42	32	62	50	10	30	22	80	62	76	56	30

Solution: here n = sample size = 400

Day	no. of defective seals (d)	$p = \frac{d}{n}$
1	28	0.0700
2	18	0.0450
3	40	0.1000
4	42	0.1050
5	32	0.0800
6	62	0.1550
7	50	0.1250
8	10	0.0250
9	30	0.0750
10	22	0.0550
11	80	0.2000
12	62	0.1550
13	76	0.1900
14	56	0.1400
15	30	0.075

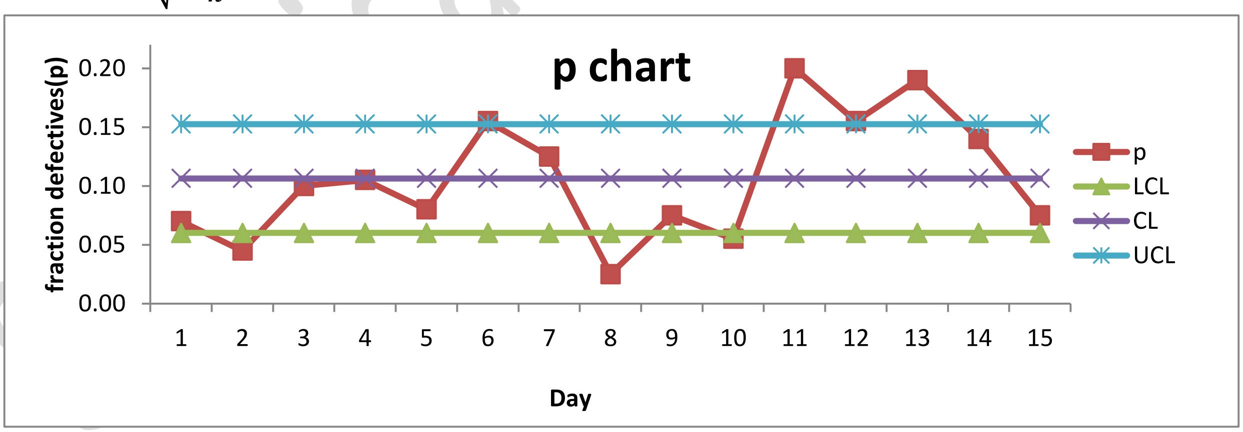
$$\overline{p} = \frac{\sum p}{m} = \frac{1.5950}{15} = 0.1063$$

The control limits for p –chart are

LCL= $\overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$ **= 0.0601**

CL= $\overline{p}$ **= 0.1063**

UCL= $\overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$ **= 0.1526**



Conclusion: The sample points of days 2, 6, 8, 10, 11, 12 and 13 falls outside the control limits. The process is therefore out of control. However the point on days 2, 8, and 10 indicates good level of quality on those days.

6 Ten samples each of 100 items are drawn from a production process. The numbers of defective items in the samples are respectively 12, 10, 0, 15, 5, 7, 13, 10, 9, and 11. Draw an appropriate chart for the given data and give your conclusion.

Solution:

The most appropriate control charts for the given data is np chart.

sample no.	no. of defectives (d)
1	12
2	10
3	0
4	15
5	5
6	7
7	13
8	10
9	9
10	11
Total	92

m = no. of samples = 10

n = sample size = 100

$n\bar{p} = \frac{\sum d}{m} = \frac{92}{10} = 9.2$

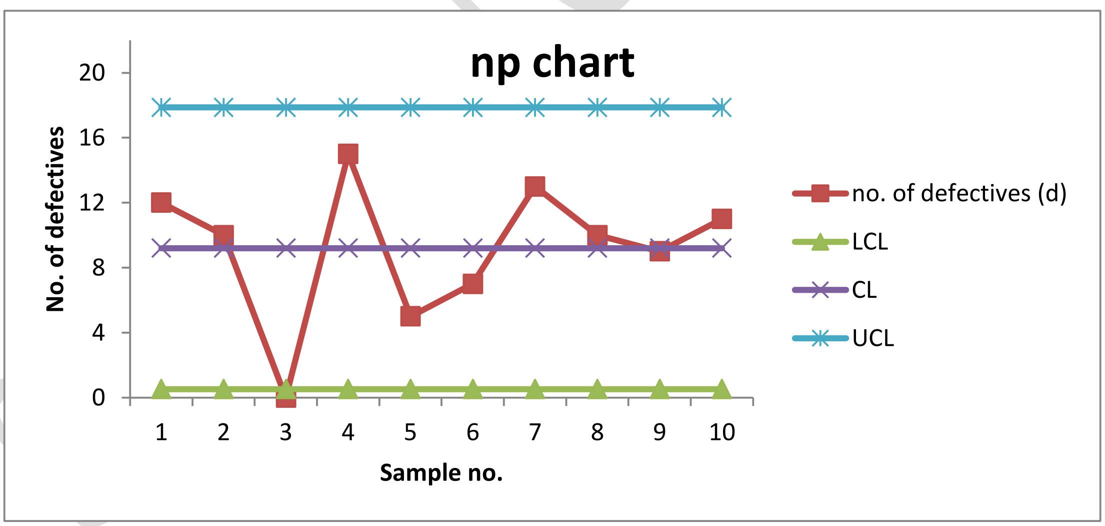
$\bar{p} = \frac{n\bar{p}}{n} = \frac{9.2}{100} = 0.092$

The control limits for np – chart:

LCL= $n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$ = 0.5292

CL= $n\bar{p}$ = 9.2000

UCL= $n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$ =17.8708



Conclusion: Since all the points fall within the control limits hence process is under control.

7 **The following table shows the number of missing rivets observed at the time of the inspection of 12 aircrafts. Find the control limits for the number of defects chart and comment on it**

OR

The following table shows the number of missing rivets observed at the time of the inspection of 12 aircrafts. Draw an appropriate control charts comment on it

Aircraft No.	1	2	3	4	5	6	7	8	9	10	11	12
No. of missing rivets	7	15	13	18	10	14	13	10	20	11	22	15

Solution:

The appropriate chart for the given data is C chart.

Here m = no. of samples = no. of aircrafts = 12

Aircraft No.	1	2	3	4	5	6	7	8	9	10	11	12	Total
No. of missing rivets	7	15	13	18	10	14	13	10	20	11	22	15	$\sum C$ =168

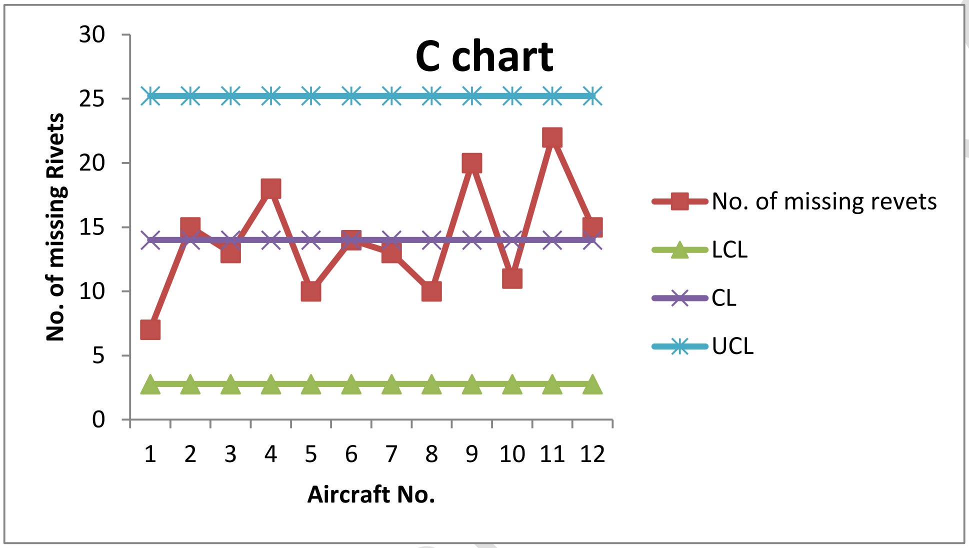
$\overline{C} = \frac{\sum C}{m} = \frac{168}{12} = 14.$

The control limits for C chart will be as under:

LCL= $\overline{C} - 3\sqrt{\overline{C}}$ **= 2.7750**

CL= $\overline{C}$ **= 14**

UCL= $\overline{C} + 3\sqrt{\overline{C}}$ **= 25.2250**



Conclusion: Since all the points fall within the control limits hence process is under control.